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Modified Monte Carlo Technique for Clausing Factor Calculations

The problem of rarefied gas flow through a cylindrical channel was first attempted by Knudsen [1] by equating the rate of momentum transfer to the channel wall resulted from molecular impacts to the difference in the pressure forces at the ends of the channel. The resultant analytical expression is valid only for very long channels [2]. Dushman [3] modified Knudsen's long-tube rarefied gas flow formula by adding the effect of orifice flow-resistance. Clausing [4] derived an integral equation for rarefied particle transport through a cylindrical channel and solved the equation for the case of a diffusely reflective channel surface. His result is commonly known as the Clausing Factor which, strictly speaking, is the probability that a well-thermalized gas molecule entering the channel will succeed in getting out of channel from the other end. In a more practical term, it is a factor whose product with the orifice conductance of the same radius as the channel will yield the true flow conductance of the channel. With the aid of computer, DeMarcus [2] employed several advanced numerical techniques to solve Clausing's integral equation and claimed his result was exact. However, the integral equation is quite difficult to apply to a noncylindrical channel and to account for complicated physical processes such as the intermolecular collision, a rough channel-wall surface, and the inlet surface reflection as in the case of sublimation. In principle, a straightforward Monte Carlo method could be applied to this problem. Davis [5] and Smith and Lewin [6] had tried Monte Carlo techniques successfully. Unfortunately, for long channel and more involved gas-surface interactions, the accuracy will deteriorate due to poor statistics. This difficulty is similar to the deep penetration of nuclear radiation through a thick shield.

In this note, a modified Monte Carlo method is discussed. Here, the statistical fluctuation due to the random number selection is bypassed by using the expected value of the known probability distribution function for all physical processes. The description of the method follows.

The inlet channel cross-sectional area, be it the subliming surface or the orifice of a large rarefied gas reservoir, is divided into many equal-width concentric rings. Each ring due to azimuthal symmetry is considered as a point source emitting Maxwellian molecules at the mean radius with a source strength proportional to the ring area. The angular distribution of molecules emitting from a point source

TABLE I

Comparisons of Clausing Factors Computed by Various Techniques

L/R	K (Clausing)	Q^a (DeMarcus)	P^b (Davis)	K–M (This Work)
0.5	0.8013	0.80127	0.804	0.80170
1.0	0.6720	0.67198	0.678	0.67192
1.5	0.5810	0.58148	0.571	0.58165
2.0	0.5136	0.51423	0.522	0.51486
3.0	0.4205	0.42006	0.425	0.41974
4.0	0.3589	0.35658	0.361	0.35619
5.0	0.3146	0.31053	0.315	0.31074

[a] Variational technique, claimed to be exact (See Ref. [2]).

[b] Monte Carlo technique, $\sim 10\,000$ histories, standard deviation ~ 0.005 (see Ref. [5]).

is divided into difference cones. The intensity of each cone is uniform about the azimuthal angle and falls off as a cosine function of cone angle. The fate of molecules in each cone (such as how far downstream do they impinge the channel wall?) is geometrically determined by the intersection of the space curve of the cone and the channel surface. The channel is divided into equal-width bands so that the fraction of the molecules in each cone impinging on each band can be calculated from the fraction of the space curve traced on each band. The channel wall surface collision density due to the "virgin" molecules (source molecules suffer no previous collision with the channel wall) as a function of the downstream distance is obtained by summing up the contributions of all the cones from a point source and then all the point sources in the inlet orifice. At the steady state, all molecules that collide with the channel wall will be reflected diffusely. A molecule diffusely reflected from the wall at $X + dX$ distance downstream will have a definite probability that it will leave the exit or return to the inlet or collide with other parts of the channel wall. A probability table is first constructed for molecules reflected from all downstream distances, i.e., all channel bands. With this probability table, the eventual fates of the molecules that collide with the wall (they either return to the inlet or leave the outlet) are obtained by iteratively applying the probability table to the residual surface collision density. The total number of molecules that collide with the wall, because of the leakage at both ends, will decrease after each iteration. Since the distribution of the colliding molecules will approach a symmetry about the mid-downstream distance, the iteration can be terminated as soon as this symmetry is reached. Obviously, half of the remaining molecules will get out of the exit. The ratio of the total number of molecules leaving the exit to that initially entering the inlet is the Clausing Factor. Numerically, it can be shown that the Clausing Factor is indeed a function of the channel

length to the radius ratio, L/R, only. The computational logic has been coded in a FORTRAN program TUBNOF (TUBe Non-Obstruction Factor). Excellent agreement of the result with the results of Clausing, Davis and DeMarcus (exact) is shown in Table I. In addition to the good accuracy, the present method is very efficient. The computer execution time on IBM 360/65 is about ten seconds for each run. Finally, because of the inherent flexibility, the present method can be extended to include more involved gas-surface interaction, inter-molecular collisions and even the noncylindrical channels. The computation was performed at M.I.T. Computation Center, Cambridge, Massachusetts.

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